(9.)

(From the SCIENTIFIC PROCEEDINGS of the ROYAL DUBLIN SOCIETY.)

ON THE TOTAL ANNUAL HEAT RECEIVED AT EACH POINT OF THE EARTH'S SURFACE FROM THE SUN, AND ON THE AMOUNT OF THE LOSS OF THAT HEAT CAUSED BY RADIATION INTO SPACE (NEGLECTING THE EFFECT OF THE ATMOSPHERE).

BY

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THE heat received by a given surface at any instant is

$$A \cos z \, dh,$$

where

$$A = \text{a known constant},$$
$$z = \text{sun's zenith distance},$$
$$h = \text{sun's hour angle},$$

and

$$\textit{Total heat received in one day} = A \int_{\text{Sunset}}^{\text{Sunrise}} \cos z \, dh, \qquad (1)$$

Now, we have

$$\cos z = \sin\lambda \sin\delta + \cos\lambda \cos\delta \cos h,$$

where

$$\lambda = \text{latitude of place},$$
$$\delta = \text{sun's declination},$$
$$H = \text{hour of sunset}.$$

Equation (1) thus becomes—

$$\text{Total heat received in one day} = \int_{-H}^{+H} \cos z \, dh$$

$$= \int_{-H}^{+H} A \sin\lambda \sin\delta dh + \int_{-H}^{+H} A \cos\lambda \cos\delta \cos h \, dh,$$

$$= 2 A \{\sin\lambda \sin\delta \cdot H + \cos\lambda \cos\delta \sin H\}. \qquad (2)$$

But, since

$$\cos H = - \tan\lambda \tan\delta,$$

the expression (2) may be thus written:

$$\text{Total heat received in one day} = 2 A \sin\lambda \sin\delta \{H - \tan H\}. \qquad (3)$$

This expression might be expanded by means of Leibnitz' theorem, as follows:

$$\text{Total heat received in one day}$$

$$= 2 A \sin\lambda \sin\delta \left(\frac{\tan^3 H}{3} - \frac{\tan^5 H}{5} + \frac{\tan^7 H}{7} - \&c.\right)$$

But this would not answer for calculation, as H passes through 90°, at the time of the equinoxes, when $\tan H$ becomes infinite and $\sin\delta$ vanishes. I therefore expand in terms of $\cos H$ as follows:

$$H = \frac{\pi}{2} - \left\{ \cos H + \frac{1}{2}\cdot\frac{\cos^3 H}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{\cos^5 H}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{\cos^7 H}{7} + \&c. \right. \qquad (5)$$

and,

$$\tan H = \frac{\sqrt{1-\cos^2 H}}{\cos H}$$

$$= \frac{1}{\cos H}\left\{1 - \frac{\cos^2 H}{2} - \frac{1}{1\cdot 2}\cdot\frac{\cos^4 H}{2^2} - \frac{1\cdot 3}{1\cdot 2\cdot 3}\cdot\frac{\cos^6 H}{2^3} - \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3\cdot 4}\cdot\frac{\cos^8 H}{2^4} - \&c. \right. \qquad (6)$$

Therefore,

$$H - \tan H = \frac{\pi}{2} - \frac{1}{\cos H} - \frac{1}{2}\cos H - \frac{1}{24}\cos^3 H - \frac{1}{80}\cos^5 H - \frac{5}{896}\cos^7 H - \&c. \qquad (7)$$

But,

$$\cos H = -\tan\lambda\tan\delta = -\frac{\tan\lambda\sin\delta}{\sqrt{1-\sin^2\delta}}$$

$$= -\frac{\sin\lambda\sin\delta}{\cos\lambda}\left\{1 + \frac{\sin^2\delta}{2} + \frac{1\cdot 3}{1\cdot 2}\cdot\frac{\sin^4\delta}{2^2} + \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\cdot\frac{\sin^6\delta}{2^3} + \frac{1\cdot 3\cdot 5\cdot 7}{1\cdot 2\cdot 3\cdot 4}\cdot\frac{\sin^8\delta}{2^4} + \&c\right\} \qquad (8)$$

and

$$\frac{1}{\cos H} = -\frac{\cos\lambda}{\sin\lambda\sin\delta}\left\{1 - \frac{\sin^2\delta}{2} - \frac{1}{1\cdot 2}\cdot\frac{\sin^4\delta}{2^2} - \frac{1\cdot 3}{1\cdot 2\cdot 3}\cdot\frac{\text{in}^6\delta}{2^3} - \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3\cdot 4}\cdot\frac{\sin^8\delta}{2^4} - \&c.\right\} \qquad (9)$$

Hence, finally,

The heat received in one day $= 2A\sin\lambda\sin\delta\,(H - \tan H)$

$$= 2A \left| \begin{array}{l} \frac{\pi}{2}\sin\lambda\sin\delta \\ + \cos\lambda\left\{1 - \frac{\sin^2\delta}{2} - \frac{1}{1\cdot 2}\cdot\frac{\sin^4\delta}{2^2} - \frac{1\cdot 3}{1\cdot 2\cdot 3}\cdot\frac{\sin^6\delta}{2^3} - \&c.\right. \\ + \frac{\sin^2\lambda\sin^2\delta}{2\cos\lambda}\left\{1 + \frac{\sin^2\delta}{2} + \frac{1\cdot 3}{1\cdot 2}\cdot\frac{\sin^4\delta}{2^2} + \&c.\right. \\ + \frac{\sin^4\lambda\sin^4\delta}{24\cos^3\lambda}\left\{1 + \frac{3}{2}\sin^2\delta + \&c.\right. \\ + \frac{\sin^6\lambda\sin^6\delta}{80\cos^5\lambda}\left\{1 + \&c.\right. \end{array} \right\} \qquad (10)$$

Or, since

$$\sin\delta = \sin\Delta \sin l,$$

where

$$\Delta = \text{obliquity of ecliptic},$$
$$l = \text{sun's longitude},$$

$$= 2A \left[\begin{array}{l} \frac{\pi}{2} \sin\lambda \sin\Delta \cdot \sin l \\ + \cos\lambda \left\{1 - \frac{\sin^2\Delta}{4}(1-\cos 2l) - \frac{\sin^4\Delta}{64}(3 - 4\cos 2l + \cos 4l) - \right. \\ \quad \left. \frac{\sin^6\Delta}{512}(10 - 15\cos 2l + 6\cos 4l - \cos 6l) - \&c. \right. \\ + \tan\lambda \sin\lambda \left\{\frac{\sin^2\Delta}{4}(1-\cos 2l) + \frac{\sin^4\Delta}{32}(3 - 4\cos 2l + \cos 4l) + \right. \\ \quad \left. \frac{3\sin^6\Delta}{512}(10 - 4\cos 2l + 6\cos 4l - \cos 6l) + \&c. \right. \\ + \tan^3\lambda \sin\lambda \left\{\frac{\sin^4\Delta}{192}(3 - 4\cos 2l + \cos 4l) + \frac{\sin^6\Delta}{512}(10 - \&c.) + \&c. \right. \\ + \tan^5\lambda \sin\lambda \left\{\frac{\sin^6\Delta}{2560}(10 - \&c.) + \&c. \right. \\ + \&c. \end{array}\right] \quad (11)$$

We must now substitute for l the sun's longitude, on each day of the year, and add the 365 terms together; this will convert all the periodic terms of (11) into the sums of sines and cosines of arcs in arithmetical progression taken all round the circumference, and with a very small common difference.*

The periodic terms, therefore, vanish in the summation, and we obtain,

$$\text{The heat received in one year} = 2A \sin\lambda \Sigma \sin\delta(H - \tan H)$$

$$= 2A \times 365{\cdot}25 \left[\begin{array}{l} \cos\lambda \left\{1 - \frac{\sin^2\Delta}{4} - \frac{3\sin^4\Delta}{64} - \frac{5\sin^6\Delta}{256} - \&c. \right. \\ + \tan\lambda \sin\lambda \left\{\frac{\sin^2\Delta}{4} + \frac{3\sin^4\Delta}{32} + \frac{15\sin^6\Delta}{256} + \&c. \right. \\ + \tan^3\lambda \sin\lambda \left\{\frac{\sin^4\Delta}{64} + \frac{5\sin^6\Delta}{256} + \&c. \right. \\ + \tan^5\lambda \sin\lambda \left\{\frac{\sin^6\Delta}{256} + \&c. \right. \\ + \&c., \&c. \end{array}\right] \quad (12)$$

Substituting for Δ its value, 23° 28′, we obtain, finally,

* Equal to 59′ 8″ (the daily change in sun's longitude), or small multiples of that arc.

Total heat received in one year

$$= 2A \times 365 \cdot 25 \left| \begin{array}{l} 0 \cdot 95910 \cos\lambda \\ \quad + 0 \cdot 04187 \tan\lambda \sin\lambda \\ \quad\quad + 0 \cdot 00047 \tan^3\lambda \sin\lambda \\ \quad\quad\quad + 0 \cdot 000015 \tan^5\lambda \sin\lambda \\ \quad\quad\quad\quad + \&c. \end{array} \right\} \quad (13)$$

It is evident, from this equation, that when the latitude is small, the heat received in the year varies as the cosine of the latitude.

It is to be observed that equation (3), which expresses the heat received in one day, becomes illusory inside the arctic and antarctic circles, when the sun does not set; for then H (the hour of sunset) has an imaginary value. We must, therefore, compute the annual heat received inside these circles by summing the heat from the equinox till the time when the sun does not set or does not rise by equation (12); and adding the heat received during the time when the sun does not set.

If D denote the sun's declination, when he ceases to rise or set, equation (12), with D substituted for Δ, will give the heat received during the part of the year when the sun rises and sets, and the angle H is real.

To this must be added the heat received during the time when the sun never sets, which may be found as follows:

Referring to equations (1) and (2), we have, when the sun does not set,

Total heat in one day

$$= A \int_0^{2\pi} \cos z \, dh \qquad (14)$$

$$= A \int_0^{2\pi} \sin\lambda \sin\delta \, dh + A \int_0^{2\pi} \cos\lambda \cos\delta \cos h \, dh,$$

$$= 2A\pi \sin\lambda \sin\delta = 2A\pi \sin\lambda \sin\Delta \sin l.$$

This value must be summed through the time that the sun does not set; or, if a be the daily change in sun's longitude,

The total heat received during the time that the sun never sets

$$= 2A\pi \sin\lambda \sin\Delta \; \{\sin l + \sin(l+a) + \sin(l+2a)\} + \&c. \qquad (15)$$

$$= 2A \cdot \pi \sin\lambda \sin\Delta \cdot \frac{\sin\left(l + \frac{n-1}{2}a\right) \sin\frac{na}{2}}{\sin\frac{a}{2}};$$

where

$$\sin l = \frac{\sin D}{\sin \Delta},$$

and

n = number of days during which the sun does not set,

and

$a = 59' \; 8''$.

But it is evident that

$$l + \frac{n-1}{2}\delta = 90^\circ \quad q.p.;$$

and, therefore, (15) reduces to the following expression:—

$$\left.\begin{array}{r}\text{Total heat received during the}\\ \text{time that the sun never sets}\end{array}\right\} = 2A \cdot \pi \sin\lambda \sin\Delta \cdot \left(\frac{\sin \frac{na}{2}}{\sin \frac{a}{2}} \right) \qquad (16)$$

At the Pole itself, since the sun never sets, this expression, summed for half a year, gives the total heat received.

If we calculate from the Equator to the Arctic and Antarctic Circles, by equation (13), and from thence to the Poles, by equations (12) (with D for Δ) and (14), we obtain the following Table:—

TABLE showing the TOTAL HEAT received by various Latitudes from the Sun in the course of a Year.

Latitude.	Feet of Ice melted.	$\frac{A \cos\lambda}{\text{Feet of Ice.}}$	Difference.
0°	97·8	97·8	0·0
10	96·5	96·3	0·2
20	92·4	91·9	0·5
* 23 28′	86·7	—	—
30	85·9	84·7	1·2
40	77·3	74·9	2·4
50	66·8	62·9	3·9
† 52 30′	61·4	—	—
60	55·7	48·9	6·8
‡ 66 32′	46·6	—	—
70	46·3	33·4	12·9
80	41·9	17·0	24·9
90	40·5	0·0	40·5

The average thickness of ice melted over the entire surface of the globe (allowing for the greater areas of the lower latitudes), by the annual sun-heat, as deduced from the foregoing figures, is exactly 80 feet.

* Tropics of Cancer and Capricorn.
† Mean Latitude of Ireland.
‡ Arctic and Antarctic Circles.

The foregoing Table is constructed on the supposition that equal quantities of sun-heat are absorbed by the atmosphere at all zenith distances; but, although this supposition is only a first approximation, yet by comparing the total quantities of sun-heat at each latitude with the following Table of Mean Annual Temperatures, some valuable conclusions may be drawn relative to the absolute radiation of heat into space from the earth's surface regarded as a whole.

MEAN ANNUAL TEMPERATURES.*

South Latitudes.	Temperature.	North Latitudes.	Temperature.
°	°	°	°
0	80·1 F.	0	80·1 F.
10	78·7 „	10	81·0 „
20	74·7 „	20	77·6 „
30	66·7 „	30	67·6 „
40	57·9 „	40	56·5 „
50	47·8 „	50	43·4 „
60	35·3 „	60	29·3 „
		70	14·4 „
		80	4·5 „

Let

T = annual sun-heat at a given latitude measured in feet of ice;
θ = mean annual temperature of a given latitude;
k = an unknown coefficient;
R = unknown radiation into space at that latitude.

Assuming that θ, the mean annual temperature of a given parallel of latitude, is proportional to the heat *retained*, we have—

T = total heat *received;*
$k\theta$ = heat *retained;*
R = heat *lost* by radiation;

and, therefore,

$$T = k\theta + R.$$

* W. Ferrel, United States Coast Survey. "Meteorological Researches," Part I., 1877.

This gives us, in the Southern Hemisphere, the following seven equations:—

0° . . .	$97{\cdot}8 = 80{\cdot}1\,k + R.$	(1)
10 . . .	$96{\cdot}5 = 78{\cdot}7\,k + R.$	(2)
20 . . .	$92{\cdot}4 = 74{\cdot}7\,k + R.$	(3)
30 . . .	$85{\cdot}9 = 66{\cdot}7\,k + R.$	(4)
40 . . .	$77{\cdot}3 = 57{\cdot}9\,k + R.$	(5)
50 . . .	$66{\cdot}8 = 47{\cdot}8\,k + R.$	(6)
60 . . .	$55{\cdot}7 = 35{\cdot}3\,k + R.$	(7)

Any two of these equations will determine k and R; and hence we have 21 combinations for finding their values.

These all give consistent results, and the mean values of k and R, derived from the 21 combinations, are—

$$k = 0{\cdot}8995.$$
$$R = 22{\cdot}405 \text{ feet of ice.}$$

As the distribution of heat near the equator is disturbed by the motions of the heated water, so that the parallel of 10° N. is actually hotter than the equator, I have made another calculation, throwing out the latitudes 0° and 10°, which reduces the combinations (from latitudes 20° to 60°) to 10 in number. The result of this calculation is—

$$K = 0{\cdot}8512.$$
$$R = 22{\cdot}60 \text{ feet of ice.}$$

The agreement of these results with the former shows that our formula represents well the whole of the Southern Hemisphere, whose annual radiation of heat may be represented by 22 feet of ice melted.

In the Northern Hemisphere we have the following nine equations—

0° . . .	$97{\cdot}8 = 80{\cdot}1\,k + R.$	(1)
10 . . .	$96{\cdot}5 = 81{\cdot}0\,k + R.$	(2)
20 . . .	$92{\cdot}4 = 77{\cdot}6\,k + R.$	(3)
30 . . .	$85{\cdot}9 = 67{\cdot}6\,k + R.$	(4)
40 . . .	$77{\cdot}3 = 56{\cdot}5\,k + R.$	(5)
50 . . .	$66{\cdot}8 = 43{\cdot}4\,k + R.$	(6)
60 . . .	$55{\cdot}7 = 29{\cdot}3\,k + R.$	(7)
70 . . .	$46{\cdot}3 = 14{\cdot}4\,k + R.$	(8)
80 . . .	$41{\cdot}9 = 4{\cdot}5\,k + R.$	(9)

These nine equations furnish 36 combinations for finding k and R, and of these, 33 give consistent results; but three combinations, viz.:—

$$(0°-10°), \quad (0°-20°), \quad \text{and} \quad (10°-20°),$$

give results inconsistent with the others, in consequence of the cause already stated. The mean values of k and R deduced from the remaining 33 combinations are—

$$k = 0{\cdot}7285.$$
$$R = 34{\cdot}385 \text{ feet of ice.}$$

If we throw out altogether the latitudes 0° and 10°, and calculate from the remaining 21 combinations (from 20° to 80°), we find—

$$k = 0{\cdot}7141.$$
$$R = 35{\cdot}475 \text{ feet of ice.}$$

The agreement between the results calculated from all latitudes, and those found by omitting the low latitudes, is not quite so close in the Northern as in the Southern Hemisphere; but our formula is fully justified, and we are entitled to conclude that the annual heat lost by radiation in the Northern Hemisphere may be 35 feet of ice melted.

It follows, that the mean annual radiation of heat from the whole earth is equivalent to melt a coating of ice 28·5 feet in thickness; but as the sun-heat received is equivalent to 80 feet of ice, we have 51·5 feet of ice representing heat not accounted for as heat, for the mean temperature of the earth's surface is not increased.

This balance of heat is expended in two ways:—

1. It is converted into the Geological work done by rainfall and rivers.
2. It is converted into Chemical and Vital work done by the vegetable and animal organisms that clothe the surface of the earth.

The Geological work done by rainfall and rivers can be shown to absorb a very small portion of the surplus sun-heat.

The Mechanical work done in crushing to fine powder a cubic foot of rock can be estimated from the following *data*, taken from

the stamps of Polberro Tin Mine. Each stamp weighs 600 lbs., and is lifted and falls through 9 inches 45 times in one minute. Each stamp crushes into fine powder 28 cwt. of tinstuff in twenty-four hours.

Hence,

$$\left.\begin{array}{l}\text{Work done in crushing one}\\ \quad\text{cubic foot of rock}\ \ .\quad .\end{array}\right\} = 713{\cdot}5 \text{ ft. tons.}$$

But, we know that the

$$\left.\begin{array}{l}\text{Work done in melting one}\\ \quad\text{cubic foot of ice}\ \ .\quad .\end{array}\right\} = 2850{\cdot}5 \text{ ft. tons.}$$

The latter number is almost exactly four times the former, from which I conclude that

The work done in melting one cubic foot of ice would suffice to crush into powder four cubic feet of rock.

It has been shown that the Geological work done by rain and rivers takes 3,090 years to crush and carry to the sea one cubic foot of surface rock; hence we see that one foot of ice (representing sun-heat), would account for the present Geological work of 12,360 years!

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